Physics 110
Spring 2006
Fluid Mechanics – Their Solutions

1. Pressure in a fluid varies linearly with depth.
   a. What is the absolute pressure at a depth of 1000m in the ocean if the density of seawater is 1024 kg/m^3?
   b. At this depth what force must the frame around a circular submarine porthole having a diameter of 30cm exert to counterbalance the force exerted by the water?

$$a.\qquad P_d = P_{air} + \rho g h = 1.013\times10^5\ \tfrac{N}{m^2} + \left(1024\ \tfrac{kg}{m^3}\right)\left(9.8\ \tfrac{m}{s^2}\right)\left(1000m\right) = 1.0137\times10^7\ \tfrac{N}{m^2}$$

$$b.\qquad F = PA = 1.0137\times10^7\ \tfrac{N}{m^2}\times\left(\pi\left(0.30m\right)^2\right) = 2.87\times10^6\ N$$

2. A light balloon can be filled with either 400m^3 of helium (ρ_{He} = 0.179 kg/m^3) or 400m^3 of hydrogen (ρ_H = 0.0899 kg/m^3) and floated to carry a payload.
   a. If the balloon is nearly in equilibrium (meaning all accelerations are zero), what mass of a payload could a balloon filled with helium support at 0^{o}C?
   b. If the balloon is nearly in equilibrium (meaning all accelerations are zero), what mass of a payload could a balloon filled with hydrogen support at 0^{o}C?

$$\sum F_y:\qquad F_{b,air} - m_{gas} g - m_{hanging} g = m a_y = 0$$

$$\rightarrow m_{hanging} = \frac{F_{b,air} - m_{gas} g}{g} = \rho_{air} V_{air} - \rho_{gas} V_{gas} = \left(\rho_{air} - \rho_{gas}\right) V_{gas}$$

$$a.\qquad \text{for He, we have } m_{hanging} = 444.4kg$$

$$b.\qquad \text{for H, we have } m_{hanging} = 480.0kg$$

$$where\ \rho = \frac{M}{V}\ \text{and volume of air displaced by gas is the same as the volume of gas.}$$

3. The U.S. Navy has the largest warships in the world, aircraft carriers of the *Nimitz* class (for example, the *USS Ronald Regan* shown on the right.) Suppose that 50, 29,000kg airplanes take off from the flight deck and the ship bobs up to float 11cm higher in the water, in an area where g = 9.78m/s^2. What is the horizontal area enclosed by the waterline of the ship? Compare this to the deck of an aircraft which has an area 18,000 m^2.

www.aerospaceweb.org/ question/history/q0226.shtml

$M_{total,planes} = 50 \times 29{,}ooo kg = 1.45 \times 10^6 kg$

The weight of the planes is equal to the weight of a slice of water 11cm thick with cross sectional area bounded by the waterline of the ship.

$M_{water} = \rho V = \rho_{water} dA \rightarrow 1.45 \times 10^6 kg = \left(1030 \frac{kg}{m^3}\right)(0.11m)A \rightarrow A = 1.28 \times 10^4 m^2$

$\therefore ratio = \frac{A_{waterline}}{A_{flightdeck}} = 0.71$

4. A Boeing 777 has a mass of 2.43x10^5 kg and each wing has an area of 189m^2. During level flight, the pressure on the lower wing surface is 700x10^4 Pa.
    a. What is the pressure on one of the upper wings?
    b. What is the upward acceleration of the aircraft if the pressure on the lower surface were to increase to 702x10^4 Pa? (This increase in pressure is due to the aircraft increasing its forward velocity and assumes that P_{upper} remains constant.)

www.boeing.com

a. If the plane is not accelerating up or down then the difference in pressures above and below the wing gives the lifting force (when multiplied by the wing area.)

$$P_{lower} - P_{upper} = \frac{F_{lift}}{A} = \frac{m_{plane} g}{A_{wing}} \rightarrow P_{upper} = P_{lower} - \frac{m_{plane} g}{A_{wing}} = 6.987 \times 10^6 \frac{N}{m^2}$$

b. $(P_{lower} - P_{upper})A_{wing} = F_{unbalanced} \rightarrow F_{unbalanced} = 6.73 \times 10^6 N$

$\therefore F_{unbalanced} - m_{plane} g = m_{plane} a \rightarrow a = 15.8 \frac{m}{s^2}.$

5. A helium filled balloon is tied to a 2.0m long string where the mass of the string is 0.05kg. The balloon is spherical with radius of 0.40m. When released the balloon lifts a length of string h and then remains in equilibrium. What length of the string is lifted if the balloon has a mass of 0.250kg?

From a free body diagram we find

$F_{air} - F_{g,balloon} - F_{g,He} - F_{g,string} = ma_y = 0 \rightarrow$

$0 = \rho_{air} gV - m_{balloon} g - \rho_{He} gV - m_{string} g \frac{h}{L}$

$\therefore h = \frac{(\rho_{air} - \rho_{He})V - m_{balloon}}{m_{string}} L = 1.91m$

6. A uniform disk with a mass of 10kg and a radius of 0.250m spins at 300 rpm on a low-friction axle. The disk must be brought to rest in 1.0min by a brake pad that makes contact with the disk at an average distance of 0.220m from the axis. The coefficient of friction between the brake pad and the disk is 0.5. A piston

in a cylinder with a diameter of 5.0cm presses the brake pad against the disk. What is the pressure of the brake fluid in the cylinder?

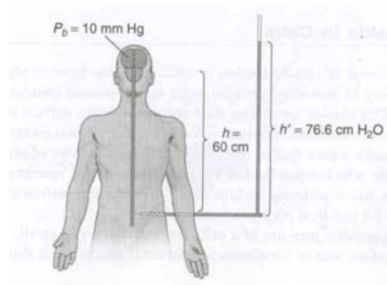
$$\omega_i = 300\,\tfrac{rev}{\min} = 31.42\,\tfrac{rad}{s} \rightarrow \omega_f = \omega_i + \alpha t \rightarrow \alpha = -0.524\,\tfrac{rad}{s^2}$$

$$\tau = rF_{friction} = I\alpha \rightarrow F_{friction} = \frac{I\alpha}{r} = \frac{\left(\frac{1}{2}MR^2\right)\alpha}{r} = -0.744N$$

$$F_{friction} = \mu_K F_{applied} \rightarrow F_{applied} = \frac{F_{friction}}{\mu_K} = -1.49N$$

$$\therefore P = \frac{F_{applied}}{A} = \frac{1.49N}{\pi(0.05m)^2} = 189.5\,\tfrac{N}{m^2}$$

7. Suppose that the pressure in the cerebrospinal fluid (CSF) is measured as shown below, using a spinal tap with the patient sitting erect. The pressure due to the weight of the CSF (given that its density is 1050 kg/m^3) in the spinal column increases the pressure?
    a. What is the pressure measured in Pascal and in centimeters of water if the pressure around the brain is 10mm Hg and the tap is at a point 60cm lower than the brain?
    b. What is the pressure measured in Pascal and in centimeters of water if the person is lying down?



a. $$P = P_{brain} + \rho g h = 10mmHg \times \frac{133.3\,\frac{N}{m^2}}{1mmHg} + \left(1050\,\tfrac{kg}{m^3} \times 9.8\,\tfrac{m}{s^2} \times 0.6m\right) = 7507\,\tfrac{N}{m^2} = 56.3mmHg$$

$$\text{height of water}: P = \rho g_{water} h_{water} \rightarrow h_{water} = \frac{P}{\rho g_{water}} = 0.766m = 76.6cm$$

b. $$P_{lying\,down} = 1333\,\tfrac{N}{m^2} = 10mmHg$$

$$h_{water} = \frac{P}{\rho g_{water}} = 0.136m = 13.6cm$$

8. The total flow rate of the adult circulatory system is about 10^{-4} m^3/s. This is the flow that passes through either the right or left side of the heart. The aorta, which carries the blood from the left side of the heart, has a diameter of 2.5cm. What is the flow velocity of the aorta?

$$flow\ rate = Av \rightarrow v = \frac{flow\,rate}{A} = \frac{1\times10^{-4}\,\frac{m^3}{s}}{\pi(0.0125m)^2} = 0.204\,\tfrac{m}{s} = 20.4\,\tfrac{cm}{s}$$

9. Suppose that the aorta has a radius of about 1.25cm and that the typical blood velocity is around 30 cm/s and that it has an average density of 1060 kg/m^3.

a. What is the average blood velocity in the major arteries if the total cross sectional area of the major arteries is 20cm^2?
b. What is the total flow rate?
c. On the assumption that all the blood in the circulatory system goes through the capillaries, what is the total cross sectional area of the capillaries if the average velocity of the blood in the capillaries is 0.03cm/s?
d. If a typical capillary has a cross sectional area of 3x10^{-11} m^2, about how many capillaries are there in the human body?
e. What are the kinetic energy per unit volume for blood in the aorta, the major arteries, and the capillaries?
f. If a capillary has an average length of 0.75mm what is the average time that a red blood cell remains in a capillary?

$$a. \qquad v_{arteries} = \frac{flow\,rate}{A_{arteries}} = \frac{1.47\times10^{-4}\,\frac{m^3}{s}}{0.002m^2} = 0.0735\,\tfrac{m}{s} = 7.35\,\tfrac{cm}{s}$$

$$b. \qquad flow\,rate = 1.47\times10^{-4}\,\tfrac{m^3}{s}$$

$$c. \qquad flow\,rate = A_{cap}v_{cap} \rightarrow A_{cap} = \frac{1.47\times10^{-4}\,\frac{m^3}{s}}{3\times10^{-4}\,\frac{m}{s}} = 0.49m^2$$

$$d. \qquad \#capillaries = \frac{0.49m^2}{3\times10^{-11}m^2} = 1.6\times10^{10} = 16billion$$

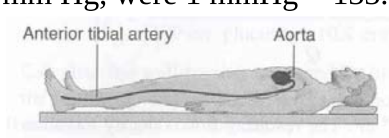
$$e. \qquad KE_{aorta} = \tfrac{1}{2}\rho_{blood}v_{aorta}^2 = 47.7\,\tfrac{J}{m^3}$$

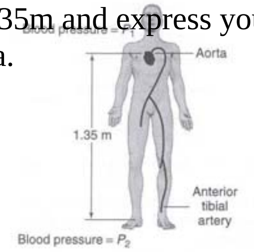
$$KE_{arteries} = \tfrac{1}{2}\rho_{blood}v_{arteries}^2 = 2.9\,\tfrac{J}{m^3}$$

$$KE_{capillaries} = \tfrac{1}{2}\rho_{blood}v_{capillaires}^2 = 4.8\times10^{-5}\,\tfrac{J}{m^3}$$

$$f. \qquad v = \frac{d}{t} \rightarrow t = \frac{d}{v} = \frac{0.75\times10^{-3}m}{3.0\times10^{-4}\,\frac{m}{s}} = 2.5s$$

10. Estimate the amount by which the blood pressure P_2 in the anterior tibial artery at the top of the foot exceeds the blood pressure P_1 in the aorta at the heart when the body is reclining horizontally and when standing erect. Assume that the distance between the aorta and the artery in the top of the foot is 1.35m and express your answer in Pascal and in mm Hg, were 1 mmHg = 133.3 Pa.





a. reclining $\rightarrow \Delta h = 0;$ from #9 $v_{aorta} = 0.30\,\frac{m}{s}$ and $v_{artery} = 0.0735\,\frac{m}{s}$

$$P_{aorta} + \tfrac{1}{2}\rho_{blood}v_{aorta}^2 = P_{artery} + \tfrac{1}{2}\rho_{blood}v_{artery}^2 \rightarrow P_{aorta} - P_{artery} = \tfrac{1}{2}\rho_{blood}\left(v_{artery}^2 - v_{aorta}^2\right) =$$

$$-44.7\,\frac{N}{m^2} = -0.34mmHg \sim 0mmHg$$

b. standing $\rightarrow \Delta h = 1.35m;$ from #9 $v_{aorta} = 0.30\,\frac{m}{s}$ and $v_{artery} = 0.0735\,\frac{m}{s}$

$$P_{aorta} + \tfrac{1}{2}\rho_{blood}v_{aorta}^2 + \rho_{blood}gh_{aorta} = P_{artery} + \tfrac{1}{2}\rho_{blood}v_{artery}^2 + \rho_{blood}gh_{artery} \rightarrow$$

$$P_{aorta} - P_{artery} = \tfrac{1}{2}\rho_{blood}\left(v_{artery}^2 - v_{aorta}^2\right) + \rho_{blood}g\Delta h = 10468.5\,\frac{N}{m^2} = 78.5mmHg$$